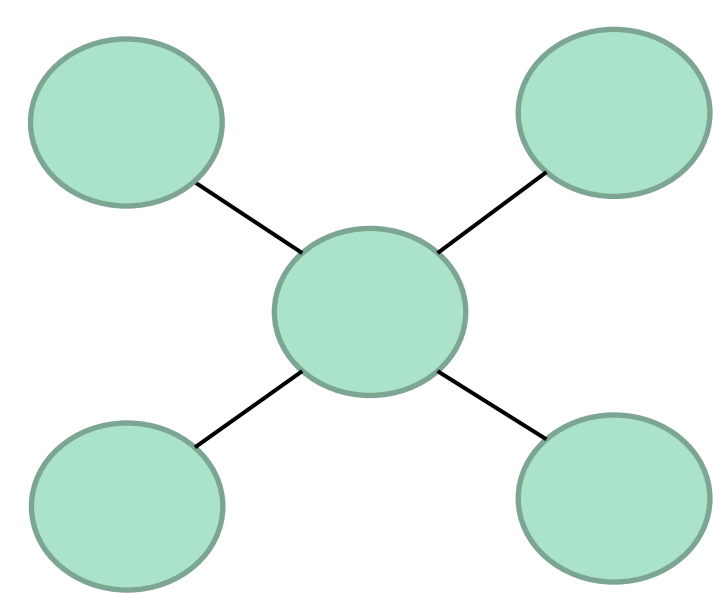


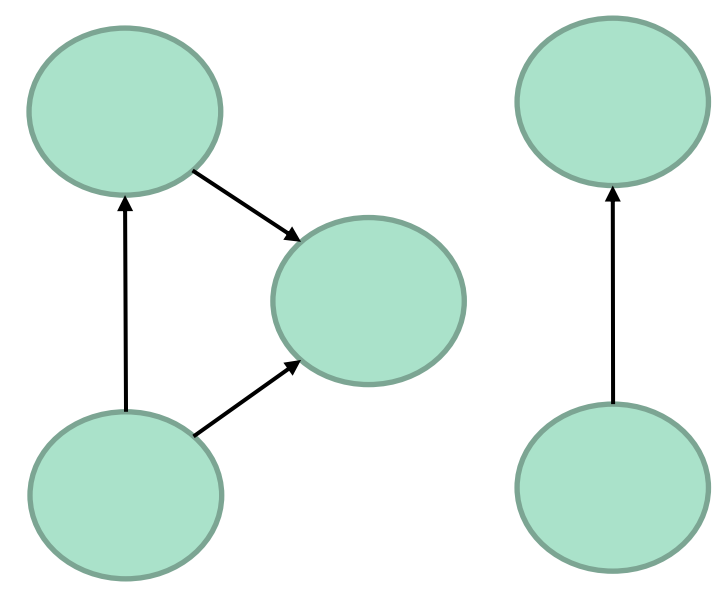
## Causality for Seismic Data Assimilation

## Beyond correlation-based data assimilation:

- For reliable risk assessment, **robust and large-scale high-performance computing** of earthquake **finite element method (FEM)** simulations is needed.
- The **robustness and predictive accuracy** of such simulations improve substantially when coupled with real-world observations through **data assimilation** techniques.
- Conventional data assimilation schemes often rely on **correlation-based coupling mechanisms** that can misrepresent the inherently **directional nature of physical interactions** in seismic wave propagation.



**Correlation modeling:**  
Undirected associations



**Causal inference:**  
Directional cause-effect  
relations to reflect signal  
propagation

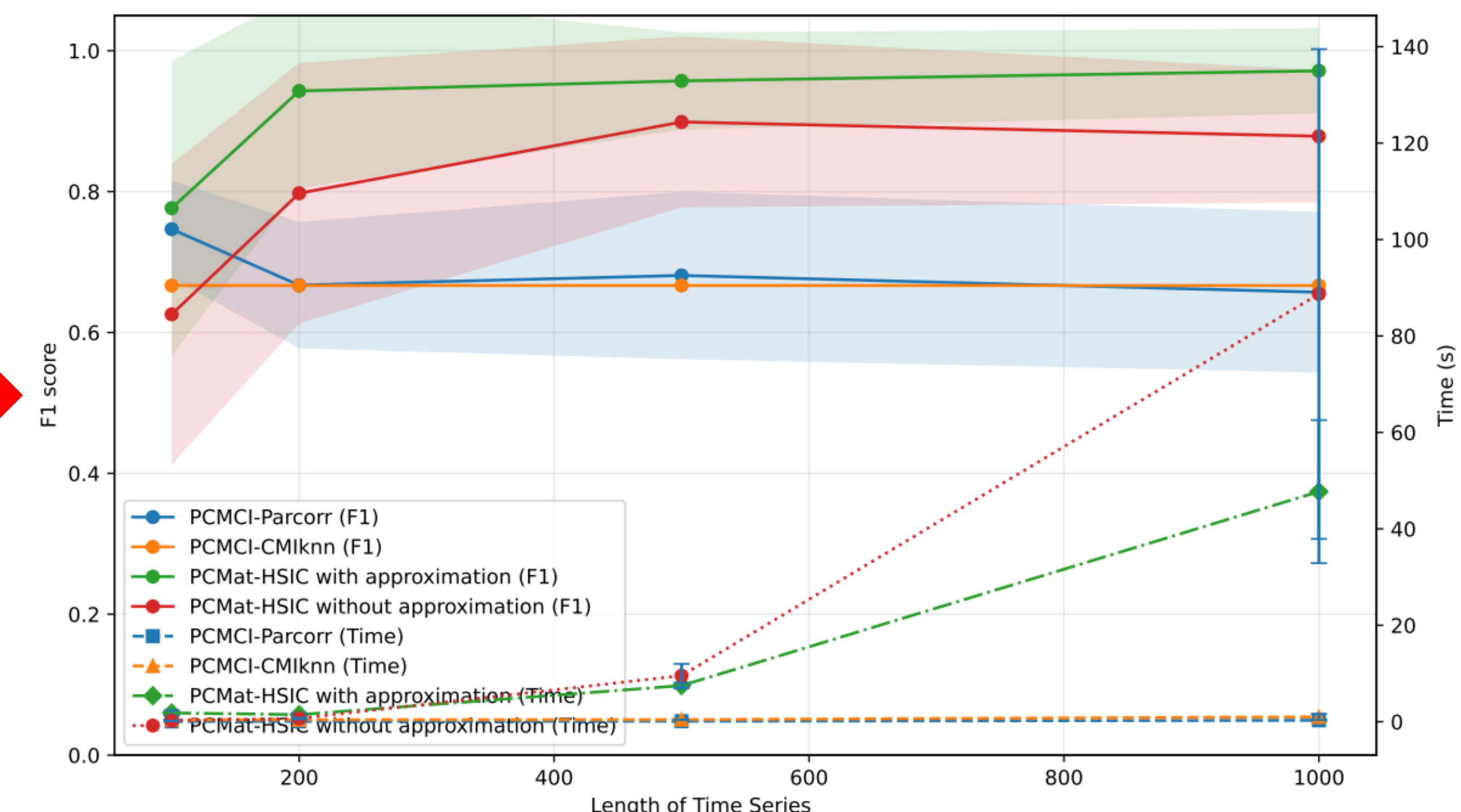
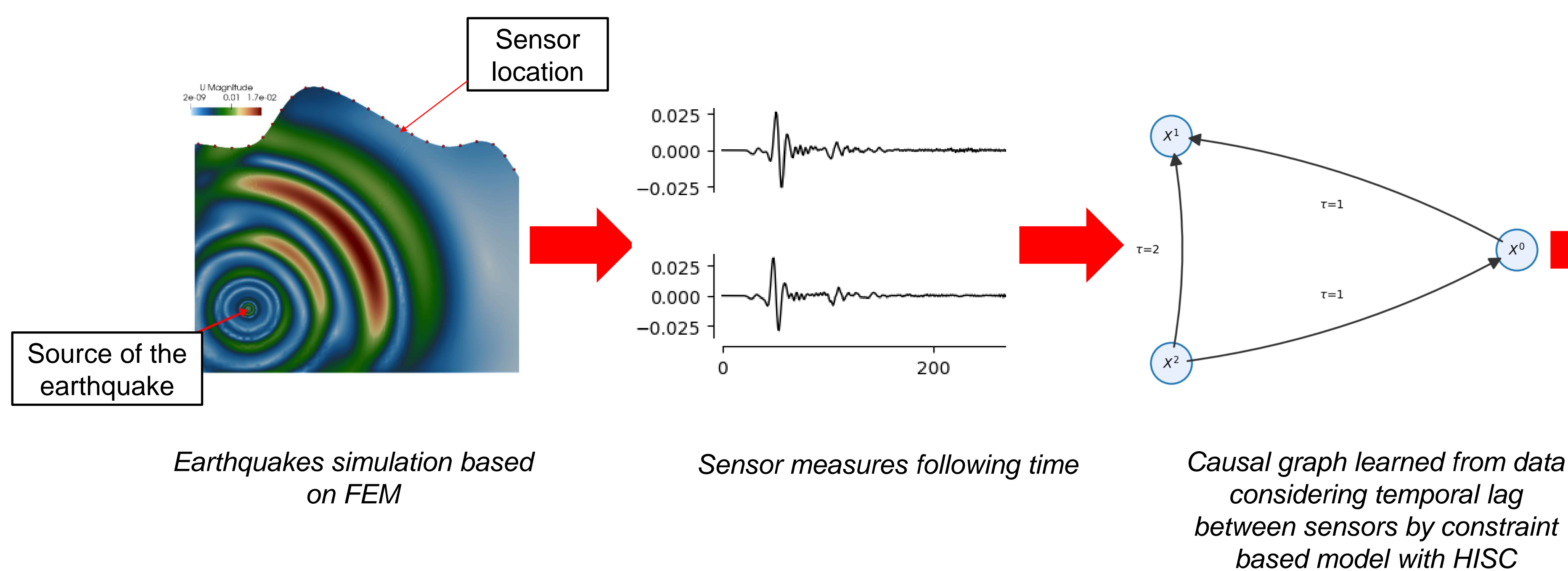
**Fig 1. Correlation-based modeling vs directional cause-effect relations in seismic wave propagation.**

## Causal discovery for time series

## Causal inference identifies directional cause-effect relations among observational sensors

- To improve upon correlation based data assimilation, **causal inference methods** identify **directional cause-effect relations** among data assimilation sensors.
- Unlike correlation-based approaches, adapted **causal discovery** algorithms extract **temporal graphs** that track signal propagation and reveal internal structure with potential lags.
- For **time series data**, conditional independence tests determine whether past values of one signal predict future values of another, beyond the target's own history.
- With **minimal modeling assumptions**, this relies on nonparametric, kernel-based independence testing using the Hilbert-Schmidt Independence Criterion (HSIC).
- Complexity of HSIC is  $O(n^2) - O(n^3)$  for time and  $O(n^2)$  for memory** due to  $n \times n$  Gram matrices, limiting scalability on long time series which arises in earthquakes (with  $n$  is the length of each time series).

## From earthquake simulation to temporal causal graphs



**F1-score and computation time across time-series lengths for PCMat-HSIC (with and without random Fourier features approximation) and for PCMat-Parcorr and PCMat-CMIknn**

**Fig 3. Causal graph learning and scaling with time-series length**

Earthquake simulation based on the finite element method generates time series at sensor locations. A causal graph is learned from data by considering temporal lag between sensors using a constraint-based model with the Hilbert-Schmidt Independence Criterion. The right panel reports F1-score and computation time across time-series lengths for PCMat-HSIC (with and without random Fourier features approximation) and for PCMat-Parcorr and PCMat-CMIknn..

## Algorithmic optimization

## Low rank random Fourier features approximation:

- Towards delivering real-time causal inference over practically sized seismic sensor network, the present work proposes a new causal algorithm PCMat-HSIC and addresses its computational challenge through algorithmic and computational optimizations.
- On the algorithmic side, a **low rank random Fourier features approximation** of the kernel matrix by sampling from the Fourier spectrum reduces construction from  $O(n^2) - O(n^3)$  to  $O(nm)$  time and  $O(nm)$  memory ( $m \ll n$ ) with  $m$  the number of RFF

## Porting the inference procedure to GPU

- The Hilbert-Schmidt Independence Criterion causal inference procedure is ported to **GPU** unit using the PyTorch library.
- Speedups up to 100 folds are reported on a NVIDIA A100 chip compared to a **CPU** baseline implementation.

## Experiments and results

- Experiment:** Comparison of causality algorithms (PCMat-Parcorr, PCMat-CMIknn, PCMat-HSIC with random Fourier features approximation, PCMat-HSIC without approximation) using F1-score (score to recover the correct links of the causal graphs) and runtime across series lengths.
- Results (Figure):** **PCMat-HSIC with random Fourier features approximation shows high F1-score with reduced runtime** relative to PCMat-HSIC without approximation as the length of time series increases; PCMat-Parcorr and PCMat-CMIknn show lower F1-score across the tested series lengths.

## Conclusion and future work

- The present work proposes a **new causal algorithm PCMat-HSIC** and addresses its **computational challenge through algorithmic and computational optimizations**.
- Future work:**
  - Integration of the random Fourier features approximation into the graphics processing unit port in order to deliver real-time inference over practically sized sensor networks
  - Comparison with other data assimilation methods.