Adaptive parallel-in-time integration for hyperbolic PDEs

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Utilizing HPC systems more efficiently is pivotal for large-scale supercomputers. One of the promising new pathways is to provide mechanisms for applications to dynamically adapt to varying resources throughout their lifetime. This capability can significantly reduce carbon footprint, reducing the average job waiting times and improve overall efficiency. This work explores the possibility of dynamic resources for parallel-in-time integration applied to a hyperbolic PDE, providing a new parallel pattern researched in the context of dynamic resource utilization.

SWEET

The **Shallow Water Equation Environment for Tests (SWEET)[1]** is a barotrop that models a simplified set of equations capturing key aspects of atmospheric dynamics within a weather model's dynamical core.

Space discretization:
* Spectral methods
* Finite difference
* Spherical harmonics

Time Integration:
* Runge-Kuttas
* Semi-Lagrangian
* PinT : Parareal, PFASST,REXI
* ....

Benchmarks:
* SWE: Galewsy, Gaussian bump
* Advection
* Burgers
* ....

LibPFASST

LibPFASST[2] is a fortran library consisting the implementation of the parallel-in--time algorithm Parallel Full Approximation Scheme in Space and Time (PFASST).

PFASST uses Spectral Deferred Corrections (SDC) to first compute inexpensive, serial approximations on a coarse grid, which are then iteratively refined on a finer grid in parallel.

By parallelizing across the time dimension, PinT enables PDE solvers to scale beyond the limits of purely spatial parallelization.

DynRes

DynRes[3] is a software stack and development environment for Dynamic Resource Management in HPC based on the Dynamic Processes with PSets design[4]:
- applicable to common HPC standard such as MPI and PMIx
- covers a divers set of HPC applications and libraries
- enables fine-grained performance-aware dynamic scheduling
- achieves improvement of key system metrics, e.g. throughput and utilization

Shallow water equations on the rotating sphere representing the dynamical core

$U = (\phi\, \zeta\, \delta)^T$ = Geopotential, vorticity, divergence

$\bar{\phi}$ = Average geopotential

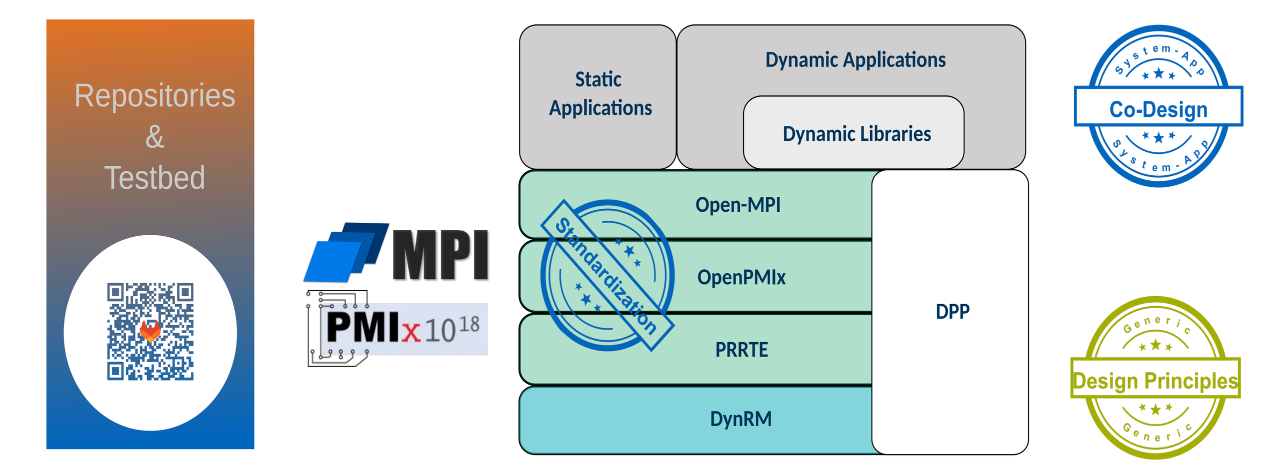
∇ = Diffusion coefficient

f = Coriolis force

V = Horizontal velocity

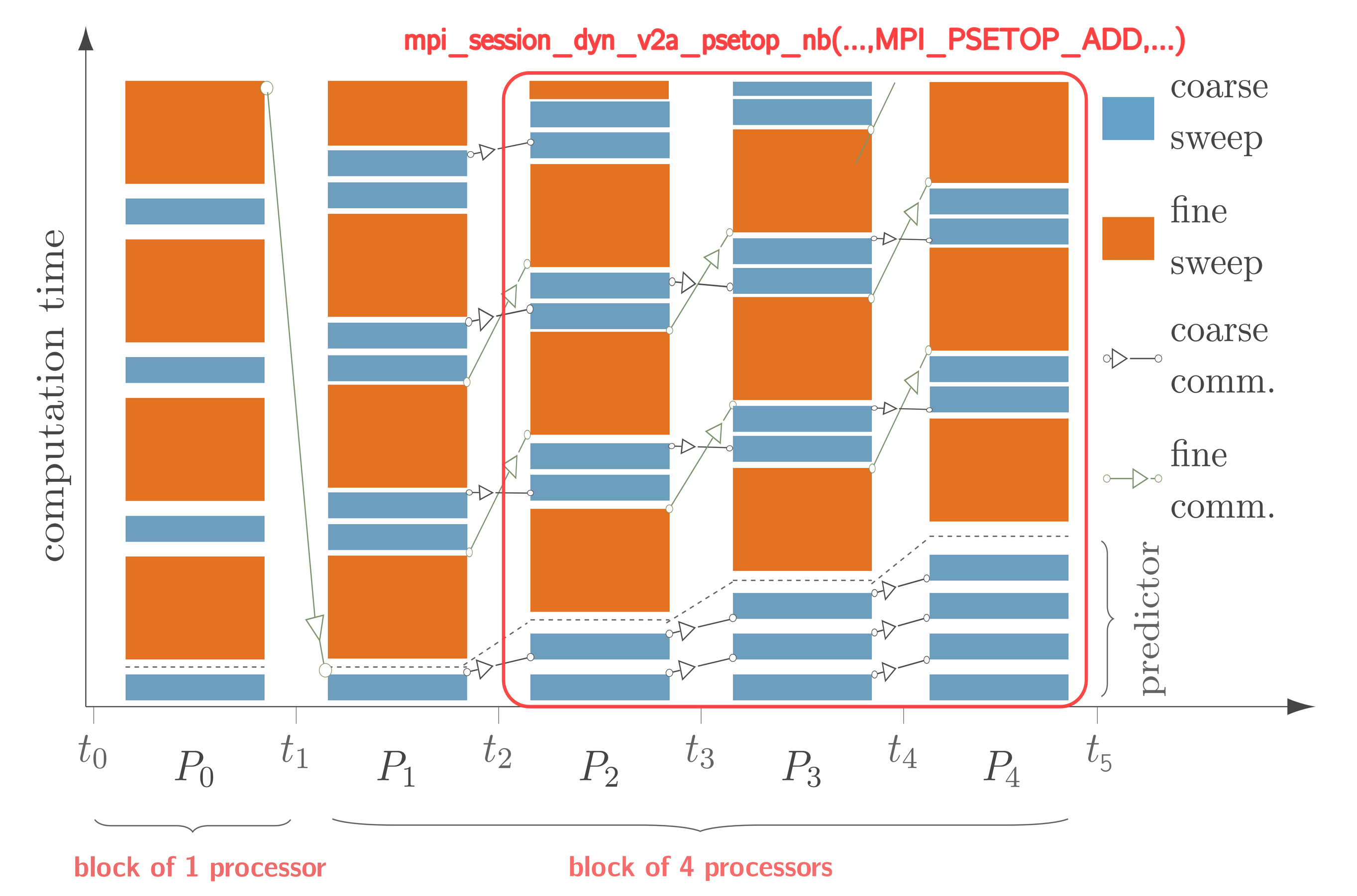
$$\frac{\partial U}{\partial t} = \underbrace{\begin{bmatrix} -\bar{\phi}\delta + \nu\nabla^2\phi' \\ \nu\nabla^2\zeta \\ -\nabla^2\phi + \nu\nabla^2\delta \end{bmatrix}}_{\text{Linear term : implicit solve}} + \underbrace{\begin{bmatrix} -\nabla\cdot(\phi' V) \\ -\nabla\cdot((\zeta+f)V) \\ \mathbf{k}\cdot\nabla\times((\zeta+f)V) - \nabla^2\left(\frac{V\cdot V}{2}\right) \end{bmatrix}}_{\text{Non-linear term : explicit solve}}$$

DynRes Software Stack



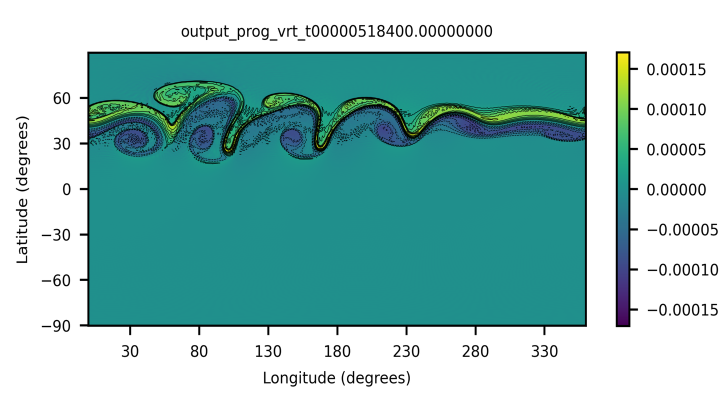
Interface implemented to integrate adaptive LibPFASST into SWEET

Convergence-informed resizing of parallel timesteps improves performance and resource utilization



Ability to add and remove processes using PSet operations and DynRes software stack

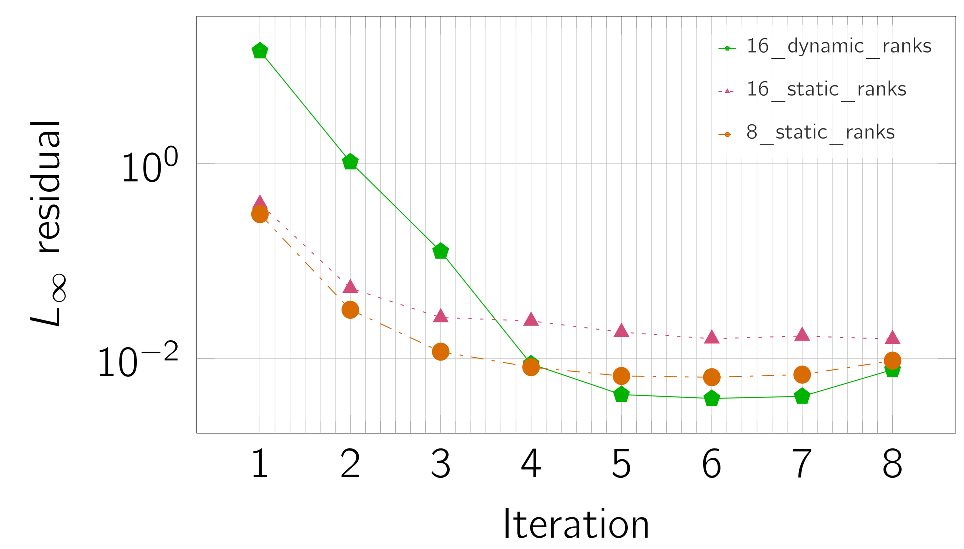
Galewsky benchmark



- Started with 8 MPI ranks on SuperMUC[3]
- 1 rank is added at every block
- Number of coarse sweeps is also increased to 2 to maintain accuracy while increasing parallelism
- Same test case run with 8 and 16 static ranks
- L-max norms used to compare static and dynamic versions
- Convergence better for the dynamic case with similar run-times!

OUTLOOK: Investigate more complex convergence-informed criteria

Parameter	Value
Spectral resolution	256 x 256
NumLevels	2
NumNodes	[5,3]
NodeType	GaussLobatto
NumSweeps	[1,1]
MPIRanks	8 to 16
T_end	18000 sec
TimeStepSize	240



[1] Martin Schreiber et al. "SWEET - Shallow Water Equation Environment for Tests v1.0". In preprint and review in Geoscientific Model Development (GMD). URL: https://egusphere.copernicus.org/preprints/2025/egusphere-2025-5156/

[2] Matthew Emmett and Michael Minion. "Toward an efficient parallel-in-time method for partial differential equations". In Communications in Applied Mathematics and Computational Science 7.1 (2012), pp. 105-132. URL: https://doi.org/10.2140/camcos.2012.7.105

[3] Dominik Huber et al. "Dynamic Resource Management in HPC systems using Dynamic Processes with PSets". In preprint in HiPC 2025. URL: https://www.martin-schreiber.info/data/publications/2025_huber_et_al_drm_in_hpc_with_psets.pdf

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